INT. J. NUM. METH. HEAT FLUID FLOW, VOL. 3, 257–266 (1993)

HEAT TRANSFER COEFFICIENT ESTIMATION BY INVERSE CONDUCTION ALGORITHM

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ABSTRACT

A method of estimating a heat transfer coefficient at the surface of a solid body is described. Knowing the ambient temperature and the temperature history at an inner point (points) of the body, the heat transfer coefficient is computed. The inverse algorithm can respect the non-linear nature of the task. The inverse algorithm is based on the computation of the temperature fields. Any method for unsteady state heat conduction can be used. The influence of the random errors of the input experimental data is described.

KEY WORDS Heat transfer coefficient Inverse conduction algorithm

NOMENCLATURE

a_1, a_2, a_3	polynom coefficients
h	heat transfer coefficient
$\mathbf{h}$	estimated value of the heat transfer coefficient
T	temperature
$T^i_{h_j}$	computed temperature in i time step using h_j
Y	experimentally established temperature
S	sum of squares of errors
M	time point where $\mathbf{h}$ is known
r_k	number of forward time steps for k estimation of the heat transfer coefficient

Superscript

i	number of time steps

Subscript

j	number of the chosen heat transfer coefficients

INTRODUCTION

The inverse heat conduction problem is the problem of determining the causes (at the surface) of measured effects within a heat conducting body. In other words, the objective of the inverse heat conduction problem is to determine the external heat transfer (the cause) given observation of the temperature history at one or more interior points (the effects). This contrasts with the usual or forward problem in heat conduction, which is to determine the internal temperature field for a given set of boundary conditions.

Most of the present work deals with the inverse heat conduction issue from Beck's method of sequential function specification[1,2]. These methods are well-established for one-dimensional

0961–5539/93/030257–10$2.00

Received April 1992
Revised November 1992

problems and for heat flux estimation. Beck's book from 1985[2] contained an extensive literature review. The latest work in inverse problems[3] points out some of the peculiar problems which do not have routine solutions. This paper should contribute to the solution for two of them: (1) heat transfer coefficient estimation, i.e. in difference to heat flux, a non-linear problem even if the nature of the direct task is linear, (2) multidimensional inverse problems.

Let us solve the inverse problem with an aim to finding the time dependent heat transfer coefficient on one surface of a solid body.

The records of temperature in n different places under the active surface are known, as well as the initial temperature distribution, the ambient temperature and material properties. A numerical method that would enable us to solve a direct problem (computation of unsteady-state temperature fields) is available. Any commonly used direct method can be used. But because of the high sensitivity of the inverse task to errors, the precision of the direct method can strongly influence the results.

The inverse algorithm, described below, can respect the non-linear nature of the heat conduction differential equation (dependent on the direct method) as well as the non-linear dependence between boundary conditions and the temperature field.

THE METHOD

The algorithm will be described using a one-dimensional case. The temperature history at an inner point is known.

Let us assume we know the temperature field up to the time step M, material properties and the ambient temperature history. Now we can compute the course of the temperature in time step $M + i$ ($i = 1, \ldots, r_k$) for the chosen value of the heat transfer coefficient. This temperature (in the point that corresponds with the position of the sensor) will be renamed $T^i_{h_j,l}$. Temperatures established experimentally are termed Y^i_l. The subscript l is for the sensor position, h for the heat transfer coefficient used in the computation and the superscript i is for the number of time steps from the time M.

The value $\mathbf{h}$ is set for the optimum estimation of the magnitude of the heat transfer coefficient in time step $M + 1$. The value $\mathbf{h}$ minimizes the least square error between the computed temperatures $T^i_{h_j,l}$ and the measured sensor temperatures Y^i_l. The least square function is:

$$S = \sum_{l=1}^{L} \sum_{i=1}^{r_k} (Y^i_l - T^i_{h_j,l})^2 w^i_l \tag{1}$$

where w^i_l is a weight that refers to the reading in sensor position l in time step i.

Differentiating S with respect to h_j, replacing h_j by $\mathbf{h}$, and setting $\partial S/\partial h_j$ equal to zero yields:

$$O = \sum_{l=1}^{L} \sum_{i=1}^{r_k} (Y^i_l - T^i_{h_j,l}) D^i_l w^i_l \tag{2}$$

where $D^i_l = \partial T^i_{h_j,l}/\partial h_j$.

Derivations of the temperature with respect to the heat transfer coefficient can be computed very precisely from the equation:

$$T = a_1 + a_2 h + a_3 h^2 \tag{3}$$

To evaluate polynom coefficients a_1, a_2, a_3, the temperature responses for three different values of h must be known. These three values h_1, h_2, h_3 are chosen from $\mathbf{h}$ taken from the previous time step: $h_1 = \mathbf{h}$, $h_2 = \mathbf{h} + \Delta h$, $h_3 = \mathbf{h} - \Delta h$. It has been numerically proved that Δh may be chosen from a wide range of values as it has no negative influence on the precision of computing D^i_l.

The temperature field in time step M can be expanded in Taylor series about the value of h_j:

$$T^i_{\mathbf{h},l} = T^i_{h_j,l} + (\mathbf{h} - h_j)\frac{\partial T^i_{h_j,l}}{\partial h_j} + \frac{(\mathbf{h} - h_j)^2}{2}\frac{\partial^2 T^i_{h_j,l}}{\partial h_j^2} + \cdots \tag{4}$$

Applying the first two parts of (4) to (2) gives:

$$O = \sum_{l=1}^{L}\sum_{i=1}^{r_k}[(Y^i_l - T^i_{h_j,l}) - (\mathbf{h} - h_j)D^i_l]D^i_l w^i_l \tag{5}$$

or

$$\mathbf{h} = \frac{\sum_{l=1}^{L}\sum_{i=1}^{r_k} h_j D^{i2}_l w^i_l + \sum_{l=1}^{L}\sum_{i=1}^{r_k}(Y^i_l - T^i_{h_j,l})D^i_l w^i_l}{\sum_{l=1}^{L}\sum_{i=1}^{r_k} D^{i2}_l w^i_l} \tag{6}$$

The inverse problem of finding the heat transfer coefficient is a non-linear one. The algorithm described above is repeated as long as the chosen value of variation of **h** is received.

The computation is carried out for $r_k > 1$ in most of the tasks. This means that results of experiments from a number of time steps are included in the computation of one step. This approach smooths the results and makes the computation more stable.

The above described algorithm can be used when several sensors are placed in different positions under the surface. Two different approaches may be used: (1) the whole process, finishing with (6), is carried out for all sensor points. The final value of **h** can be computed as a weight average of individual results. The weight of each individual result depends on the sensor data quality or dependability; (2) Beck[2] proposes to use different weights for data readings from different sensors. All experimental data are simultaneously used in (6). The weight depends on the magnitude of derivatives as follows:

$$w^i_l = \frac{D^{i2}_l}{\sum_{l=1}^{L} D^{i2}_l}, \qquad \sum_{l=1}^{L} w^i_l = 1 \tag{7}$$

The above described algorithm may be, under certain conditions, expanded to a multidimensional case. To solve a two or three-dimensional direct problem, the surface must be divided into n areas. Each area has a certain history of boundary conditions. To estimate n heat transfer coefficient histories by the inverse task, at least n sensors are necessary. Neither of the two temperature sensors can be at the same position. The sensors can be theoretically located at any position in the body. This demand is valid only theoretically. A strong dumping of the thermal wave and the limited accuracy of the measurements are the practical limits. The common demand is to measure the temperature at a reasonable distance from the active surface. The heat transfer coefficient on an arbitrary surface influences all the inner point temperatures. The reasonable position of the sensor with regard to the appropriate surface is the position where the temperature field is predominantly influenced by the appropriate estimated value of the heat transfer coefficient. In a multidimensional case, n-values of **h** applied in n surface points are computed simultaneously for each time step. The computation of all these heat transfer coefficients is relatively mutually independent and is run using the algorithm described above.

The value of **h** at any of the surface points influences the temperature field in the whole body. The mutual influence of all parameters leads to the necessity of using the iterative algorithm. The usage of the inverse task for a three-dimensional case will be shown in the final example.

TESTS

The behaviour of the method was tested on a task with triangular impulse of the heat transfer coefficient. The input data were prepared by direct computation. The material properties, sizes and distances used in this numerical test are parameters taken from a real experiment described in the following chapter. A one-dimensional body, 20 mm long, with a starting temperature of 100°C and a thermal diffusivity of 1.28×10^{-5} m^2 sec^{-1}, was exposed to the impulse of h with a maximum value of 1000 W/m^{2}K and an ambient temperature of 0°C. The opposite surface was adiabatic. The cubic spline method[4] was used to compute all direct tasks.

The results of the direct computation are used as the input data of the inverse task. The computed temperature history at a depth of 3 mm substitutes the experimental data (see *Figure 1*, the curve marked $T_{3\,\mathrm{mm}}$). All computations were made with a time step of 1 second.

The results of the inverse task are shown in *Figure 1*. This shows that the application of the described algorithm to the errorless input data gives very precise results. Different results were obtained for different numbers of forward time steps (the number r_k in (1) and (6)). The computed results become smoother as the number of the forward time steps r_k grows. The computation cannot follow the rapid changes of the heat transfer coefficient if the value of r_k is too big.

Probably the most important feature of an inverse algorithm is its sensitivity to random experiment errors. *Figures 2* and *3* show this feature for a 2% and 10% random error in the

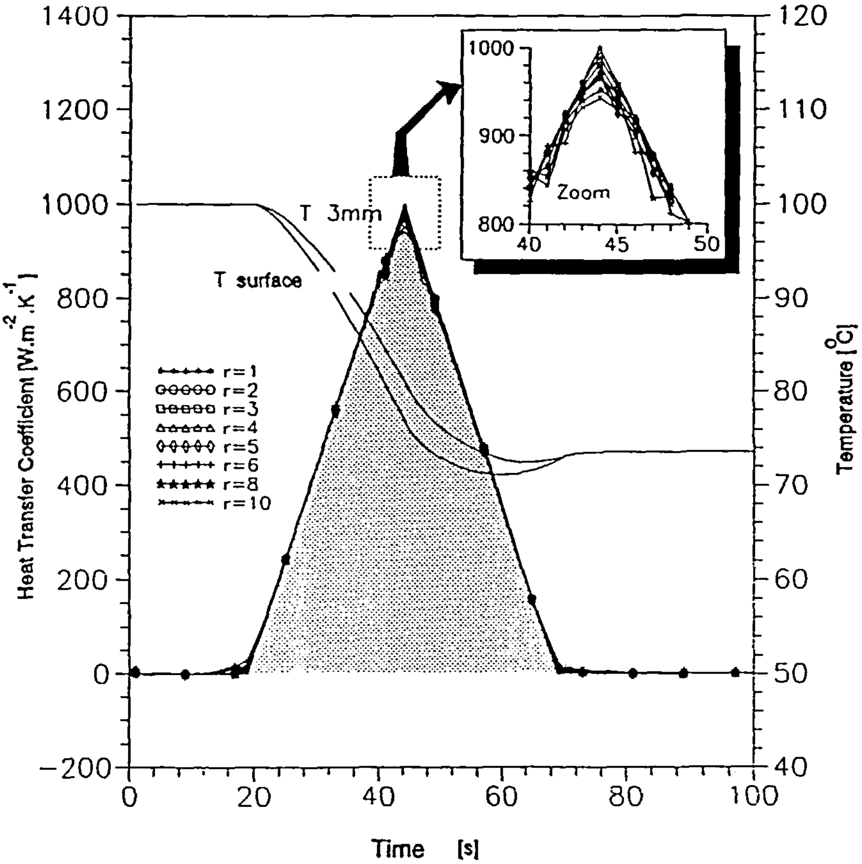

Figure 1 Triangular impulse—influence of r_k

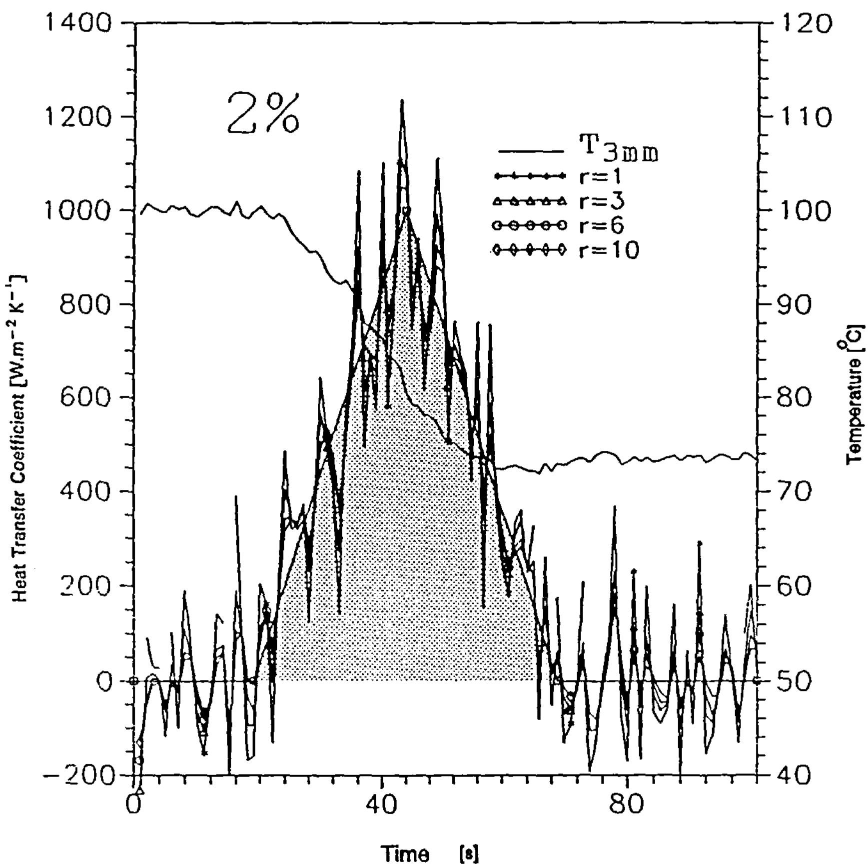

Figure 2 Influence of 2% input data random error

input data. The exact input data—the temperature history for a triangular impulse of the heat transfer coefficient at a 3 mm depth under the active surface—were deformed by adding random errors with a prescribed maximum amplitude. The results shown in *Figures 2* and *3* may be evaluated from two points of view: first, the results in both examples follow the triangular impulse. The random error of 10% debases the input data to such an extent that the results cannot seriously describe the heat transfer conditions, and secondly, choosing a bigger r_k decreases sensitivity to the random errors. The results for a bigger r_k are smoother. The method proved to be stable even for serious errors in input data.

EXPERIMENT EVALUATION

The usage of the above described inverse method for a 3-D case will be shown using an experiment evaluation. The aim of the experiment was to describe the heat transfer conditions on a surface cooled using water jets. These boundary conditions must be described for all points of the quenched surface. Ten thermocouples were built into an austenitic steel plate. The position of the thermocouples is shown in *Figure 4*. The thickness of the plate is 20 mm. The water jet is flat and narrow and is directed perpendicularly to the centre of the plate (the area of

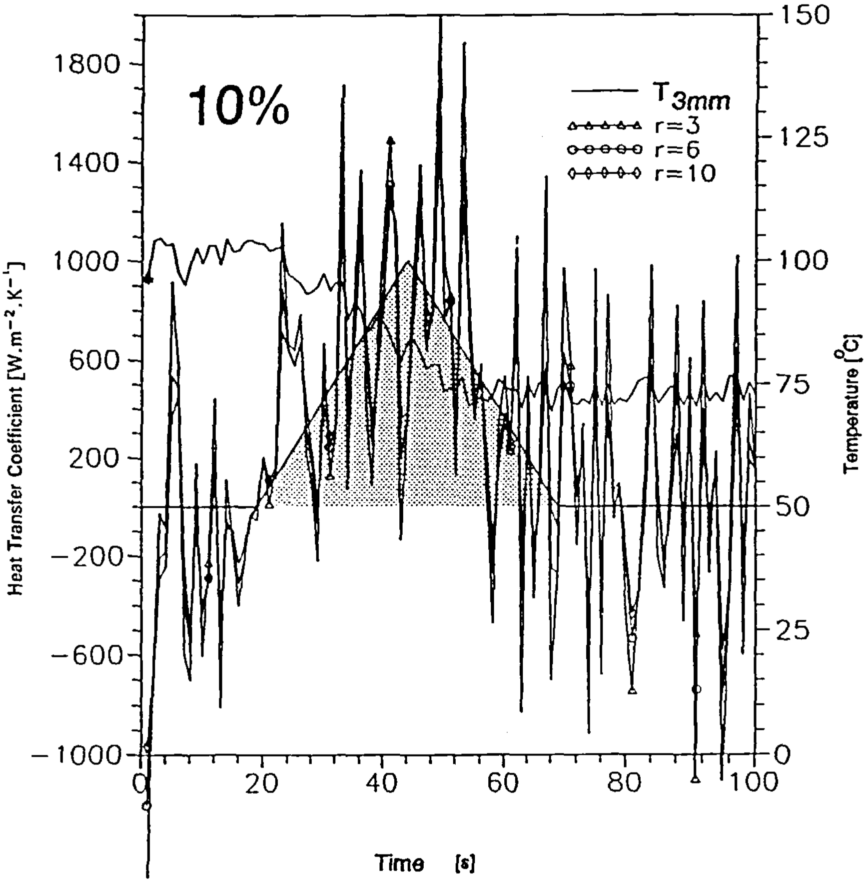

Figure 3 Influence of 10% input data random error

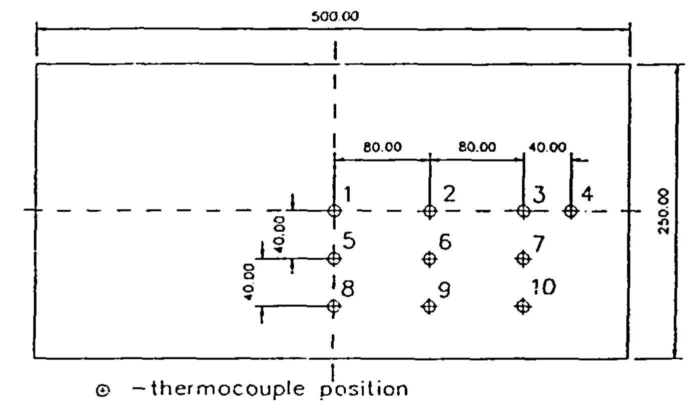

Figure 4 Testing plate and thermocouple positions

thermocouples 1–3 in *Figure 4*). The thermocouple measures the temperature at a depth of 3 mm under the active surface. The back and side surfaces of the test plate were insulated. The plate is electro-radiately heated to a uniform temperature. The maximum temperature which was used was 950°C. The cooling water is pumped from a tank to a nozzle through a flowmeter. The maximum pressure used was 1 MPa. The quenching starts at time zero. The temperature history records are stored in digital form. A typical temperature history is shown in the *Figure 5*. The data were stored with a time step of one second. These data are used as the input for an inverse task that evaluates the heat transfer conditions. The computation algorithm is as follows:

(i) Extrapolation of the starting temperature distribution for starting temperature data record.
(ii) Estimating a starting value of h at all surface points.
(iii) Solving a 3-D direct problem and using the above described inverse task to estimate **h** at the surface points referring to the sensor locations.
(iv) Interpolation and extrapolation of **h** to all surface points by the spline function.
(v) The procedure in (iii) and (iv) is repeated until the variation of **h** is bigger than the prescribed value for two time steps. This criterion is applied to all surface points.
(vi) Computing the temperature field at the time $M+1$ and repeating procedure form (iii) for the next time step.

The results for input data shown in *Figure 5* are in *Figure 6*. *Figure 6* shows the computed heat transfer coefficient history. The coefficient describes heat transfer conditions on the cooled surface.

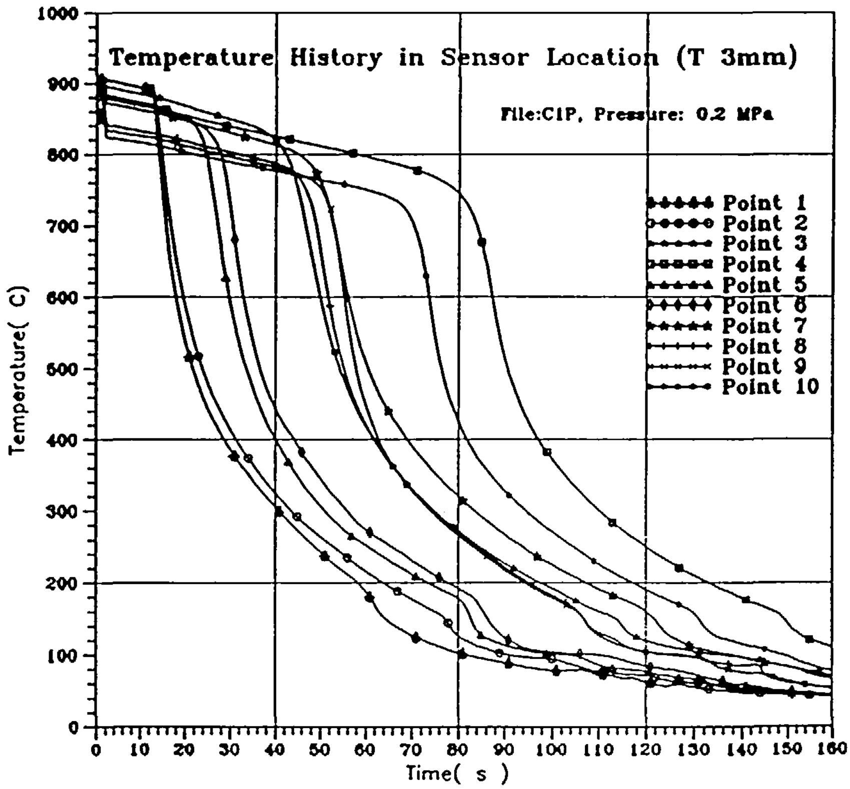

Figure 5 Temperature history in sensor positions

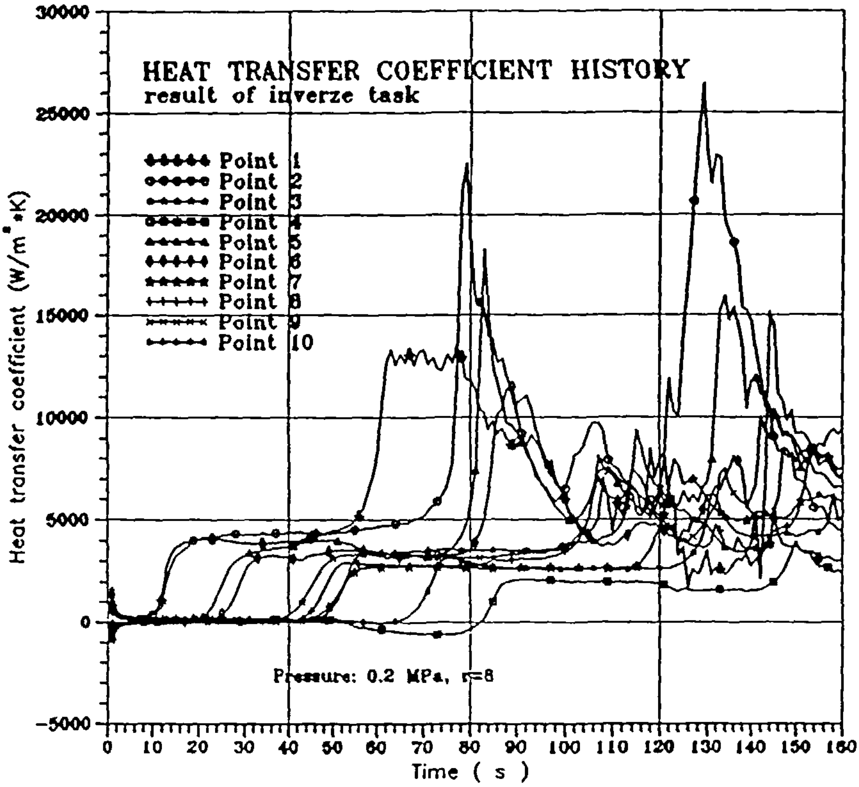

Figure 6 Computed heat transfer coefficient history

Five forward time steps ($r_k = 5$) were used for computation. The space distribution of the heat transfer coefficient above the cooled surface is shown in *Figure 7* (because it is symmetrical, only one-quarter of the plate surface is shown). Application of the inverse task allows us to describe surface temperature histories, heat fluxes and heat transfer coefficients in all points of a quenched surface.

CONCLUSIONS

The inverse method described in this paper allows us to compute the heat transfer coefficient from experimentally established data. The method can be built upon the arbitrary direct method. The non-linearity can be respected if the direct method can respect it.

If data from several sensors are available for computation of the heat transfer coefficient at one surface point, different weights should be used for these data readings. It is suitable to omit the data with very small weights. Data from sensors placed far from the active surface may decrease the stability of the computation. A very important role is played by the choice of the number of data readings used in the computation of one time step. The optimum value of r_k must ensure the numerical stability of the computation and, simultaneously, avoid smoothing the peaks in results which are caused by the physics of the studied problem. The recommended strategy for choosing r_k is to start with $r_k = 1$ and successively increase this number until the

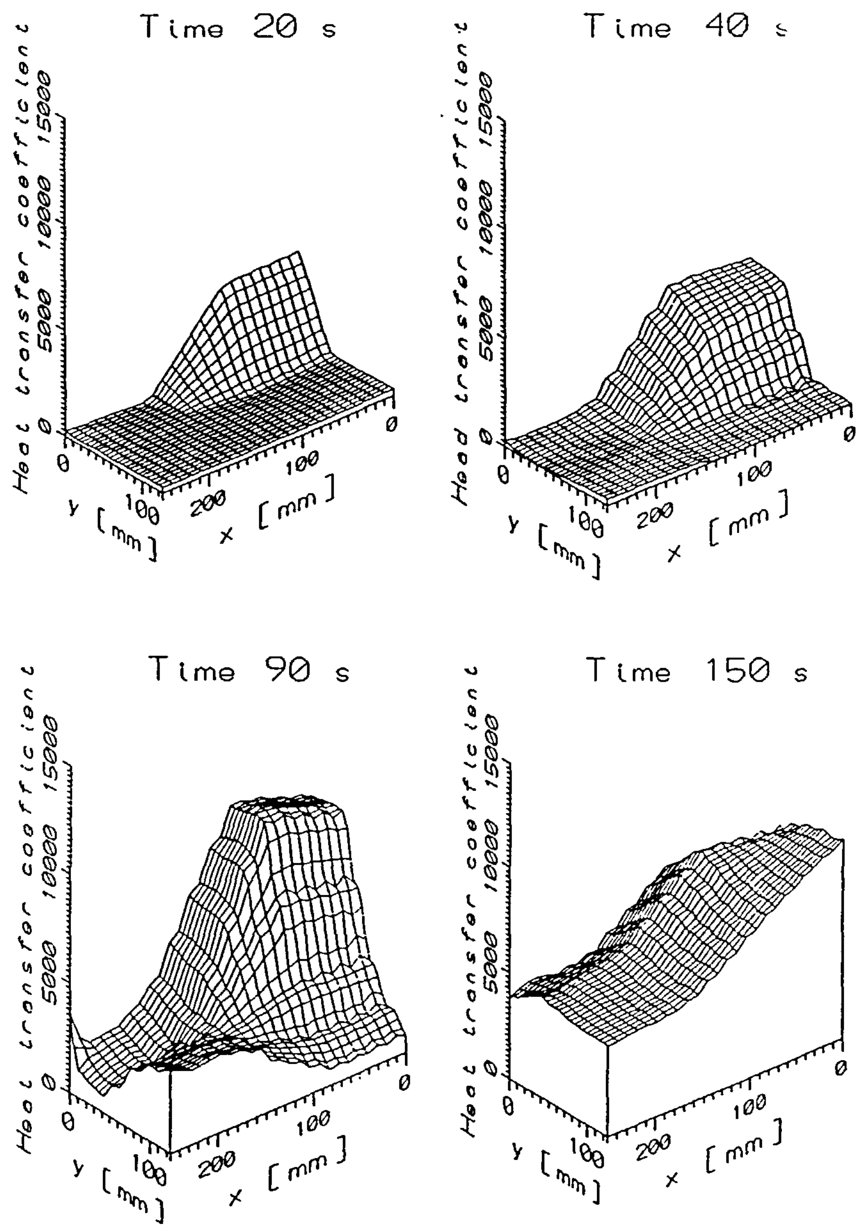

Figure 7 Heat transfer coefficient—surface distribution

stable solution is reached. This approach is especially suitable when the input data contains random errors.

The method can be used for 1-, 2- and 3-dimensional cases. The 3-D case was successfully used in technical computations. Because of the non-linear nature of the studied problem and the mutual influence of all parameters, the problem must be solved in the iterative manner (especially in multidimensional cases).

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